ACOUSTIC WAVE SCATTERING FROM A SPHERE

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I would like to start out by listing the principal objectives of our present study, and then discussing our initial efforts along these lines. The main objective is to calculate the angular and the frequency dependence of the ultrasonic energy scattered from a solid ellipsoid of revolution embedded in another solid. This calculation has to take into account mode conversion that takes place at the boundary between the ellipsoid and the medium. A second and integral part of the program is an experimental check on the calculations by way of accurate measurements of both the frequency and angular dependence of the ultrasonic energy scattered in the range of one to fifteen MHz.

Let me start out by discussing the first phase of the analysis. This first phase will concentrate on scattering from a single sphere. Dr. Cohen is leaning on a treatment by Ying and Truell,[1] which is an extension and modification of an original treatment by Herzfeld[2] in the 1930's. Dr. Cohen has independently solved the scalar and vector potential problems and has shown that his results reduce to the same results that Ying and Truell have obtained. In this treatment three types of waves are thought to arise from a longitudinal wave impinging on the obstacle: there is the wave that passes by the obstacle unimpeded by it, a second that is scattered by the obstacle, and a third which is excited inside the sphere. The latter two clearly involve mode conversion, and therefore the presence of shear waves has to be taken into account. In the treatment, the displacement and stress are assumed to be continuous across the interface between the sphere and the medium. The parameters important in the calculation are listed in Table 1 and are as follows: k_1 and K_1

are the longitudinal and shear wave vectors in the medium respectively, and K_2 and K_2 are the longitudinal and shear-wave vectors in the sphere. The n_i are the ratios between the two in the medium and the sphere separately, and Δ is the ratio of the shear modulus of the sphere and that of the medium minus one. As usual, ω is the frequency, λ and μ are Lamé's constants and ρ is the density.

The goals of the calculations are to obtain the angular and frequency dependence of the acoustic energy for both the longitudinal and shear waves. Four distinct types of spherical obstacles are being considered (as shown in Table 2): first, the cavity for which the ratio of the densities of the obstacle to that of the medium is zero; second, the rigid, motionless sphere where the ratio of densities is infinite; third, the tungsten carbide sphere in a titanium matrix; and finally, a magnesium sphere in a titanium matrix.

Case two, the rigid sphere, is more or less a check against the classical theory by Morse,[3] which considers a longitudinal wave traveling in a fluid impinging on a rigid motionless sphere. It is exactly this case where we are having difficulties. We have not been able to reproduce those results exactly. It is not clear at this time whether it is difficulty in debugging the program or whether the correct limits have been chosen for the parameters describing this case.

At any rate, to give you an idea of the type of thing that we are looking at, Fig. 1 shows some preliminary computational results for a tungsten carbide sphere inbedded in a titanium alloy. This is in the regime where the wave vector times the radius of the sphere is equal to one in the medium. The wave is to be considered incident from the left, and what you see then is the intensities (on a polar plot) of scattered compressional and shear waves. One reason I am showing this graph is to point out that the angular dependence shows a considerable angular variation for both compressional and shear waves with nulls near 90 degrees. Another reason is to show that the scattered shear wave intensity is relatively large so that interference between the compressional and shear

TABLE I

Parameters Of The Theory

k_1 - long wave vector in medium = $\omega(\frac{\lambda+2\mu}{\rho})^{-\frac{1}{2}}$

K_1 - shear wave vector in medium = $\omega(\frac{\mu}{\rho})^{-\frac{1}{2}}$

k_1 - long wave vector in sphere

K_2 - shear wave vector in sphere

$\eta_i = \frac{K_i}{k_i}$

$\Delta = \mu^{sph}/\mu^{med} - 1$

TABLE II

Types of Obstacles

(1) spherical cavity $\rho_2/\rho_1 = 0$

(2) rigid sphere $\rho_2/\rho_1 = \infty$

(3) Tungsten carbide sphere in Ti matrix

(4) Magnesium sphere in Ti matrix

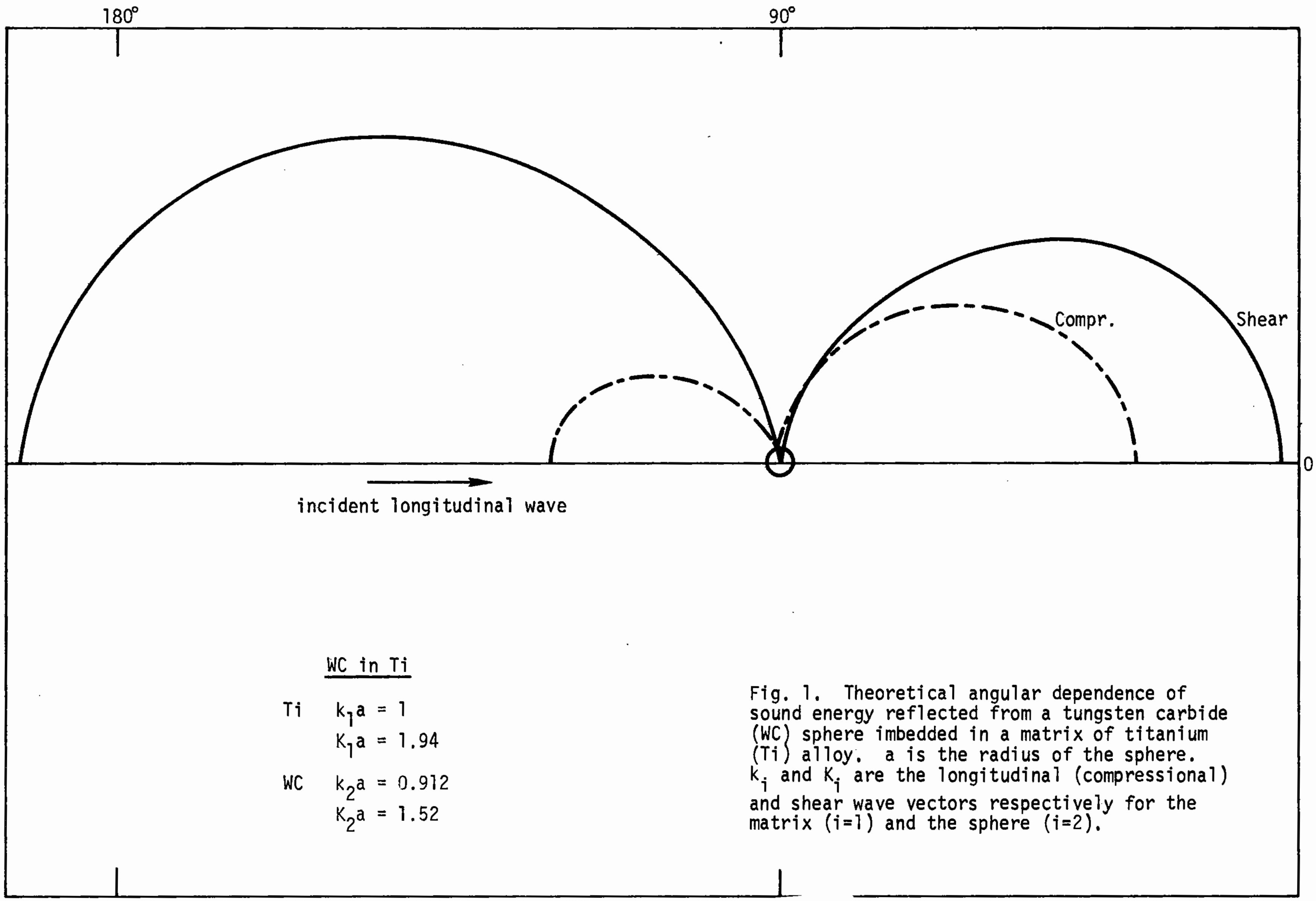

Fig. 1. Theoretical angular dependence of sound energy reflected from a tungsten carbide (WC) sphere imbedded in a matrix of titanium (Ti) alloy. a is the radius of the sphere. k_i and K_i are the longitudinal (compressional) and shear wave vectors respectively for the matrix (i=1) and the sphere (i=2).

waves must be taken into account. This feature illustrates the importance of mode conversion in the experimental regime.

Our first step in the experimental program was to decide how big to make the defect. Figure 2 is a theoretical plot of the power that is back-scattered (reflected) from the sphere as a function of the frequency and solved on the computer for the classical case described by Morse[3] for three different size defects. You can identify three regions; the region at low frequency with the typical Rayleigh scattering where the power reflected varies with the frequency to the fourth power; the region at high frequencies where the power reflected is more or less independent of the frequency, and the third region where the reflected power develops quite a bit of character and large changes in amplitude as a function of frequency. This is the region we want to exploit. It is also the region where you have to go to a computer to find the detailed nature of these curves. We decided that we would like to put this region more or less in the center of our frequency range or about 5 MHz. This dictates the diameter of the sphere to be about 400 microns.

Since a 400-μ obstacle is a very small obstacle to imbed, we decided to look very closely at the diffusion bonding process as an appropriate means of sample preparation. Figure 3 shows an example of a 400-μ hemisphere produced by the diffusion bonding process in a commercially available titanium alloy. It is a well-shaped cavity with no bond line visible in the micrograph. This sample was formed by taking two pieces of the alloy and machining each so that one was flat and the other had a hemispherical depression. The pieces were then placed into a vacuum chamber at 10^{-6} torr, heated to about 950°C, and then pressed together with 500 psi pressure. Under these conditions the two samples are bonded so that the bond line is difficult to detect acoustically or optically when the sample is cut in half, polished, etched, and inspected under the microscope. As shown in Fig. 3, grains have grown across what used to be the bond line. In this particular alloy, Ti 6%Al 4% V, two metallurgical phases are present, so a sound wave passing through the medium suffers a great deal of scattering, and for a small defect of about 400 microns,

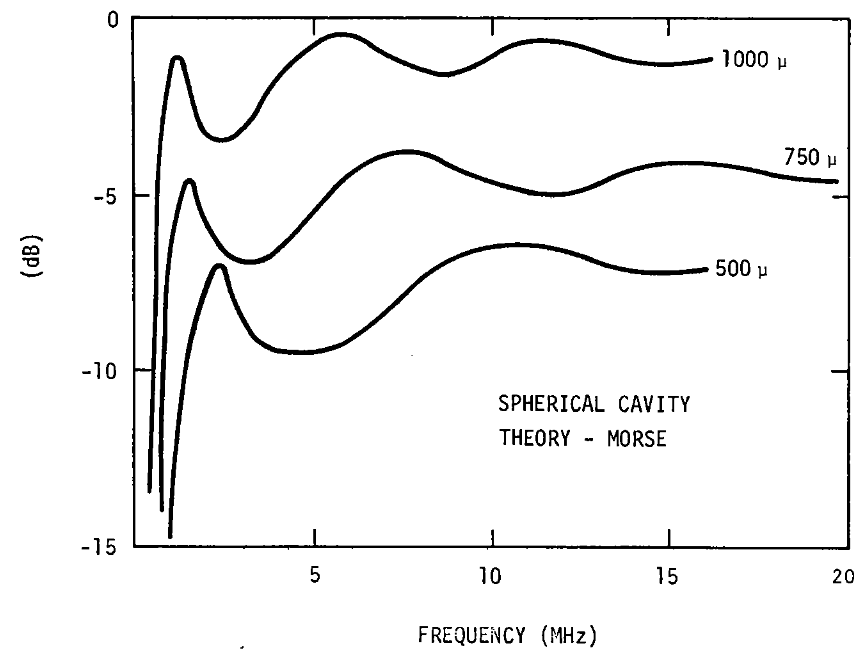

Fig. 2. Plot of theoretical (Morse[2]) reflection coefficient as a function of frequency for spheres of three different diameters (2a=1000μ, 750μ, and 500μ). The plot illustrates that a study of the frequency and amplitude dependence should in principle not only provide a recognition of the shape but also the size of a spherical cavity.

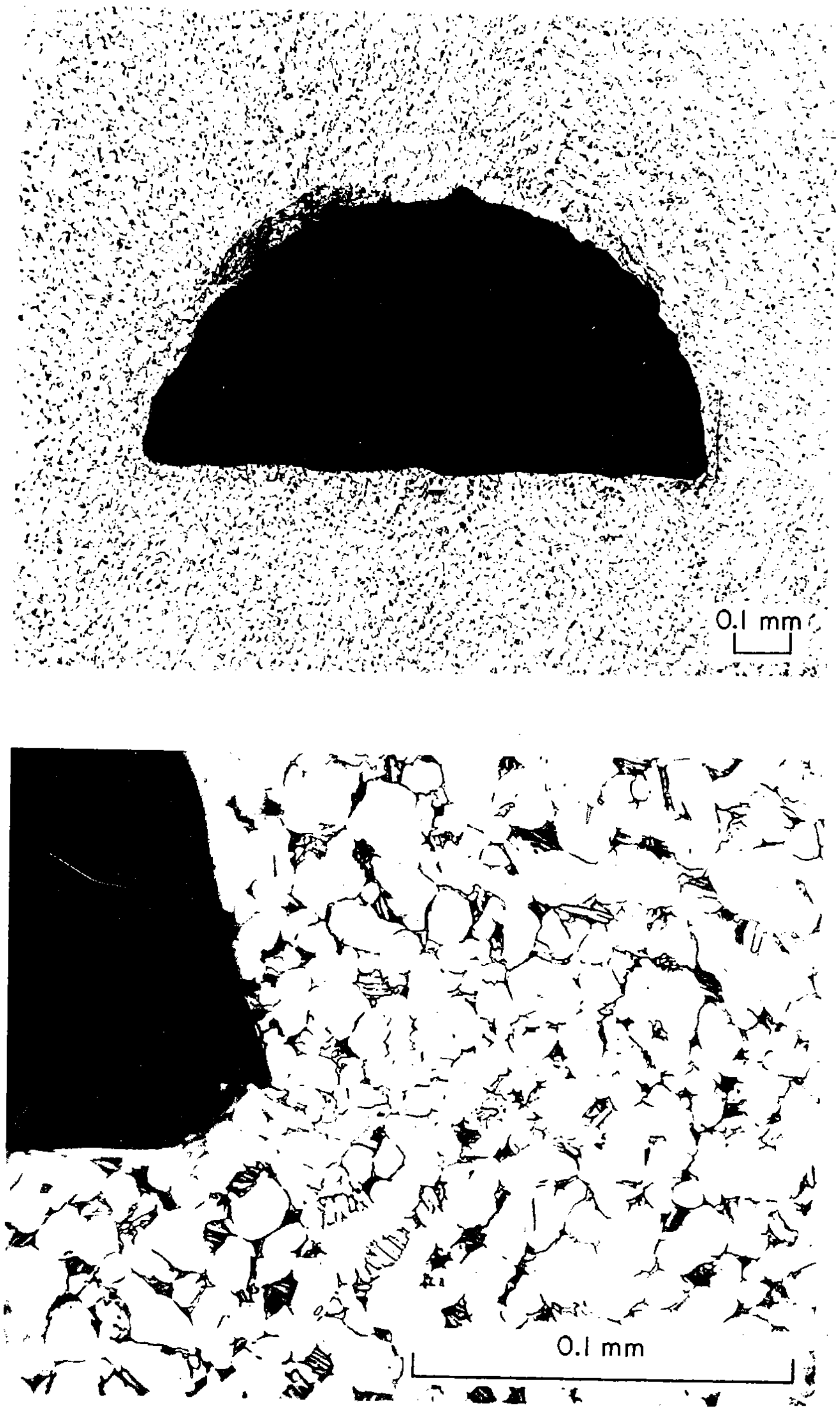

Fig. 3. Micrographs of the cross section of a hemispherical cavity produced by diffusion bonding of two machined sections of Ti-alloy. The top figure is a mosaic of several micrographs.

this will tend to make things difficult, especially when you are filling a cavity with a material that has an acoustic impedance very similar to that of the medium. We, therefore, looked around for other alloys, and one that came to mind was Ti-Al which is a single-phase alloy, but we found, and we don't fully understand why, that the attentuation and the scattering were actually somewhat higher than in the two-phase material.

The next material considered was pure titanium which, in fact, turned out to give us less scattering and attenuation, by about two db/cm at 10 MHz which, when we are talking about material depths of five centimeters or so, adds up to some ten to fifteen db, depending on the frequency, of course. The trouble with pure titanium is that it is more difficult to bond, and we are now trying to finalize the diffusion bonding parameters. We are going to use the titanium primarily for samples where the obstacles have similar acoustic impedances as the medium and leave the Ti-Al-V alloy for obstacles such as cavities or inclusions with substantially different acoustic impedances.

In the experimental program, then, we are going to look at two matrices, the pure titanium and the Ti-Al-V alloy. Table III shows the elastic constants and the density as obtained in our laboratory for the Ti-Al-V alloy. We are going to look at 400-μ and 800-μ diameter spheres and consider three scattering cases. They are a cavity which is basically a vacuum, a tungsten carbide sphere, and a magnesium sphere. The tungsten carbide sphere corresponds to an acoustic impedance in the neighborhood of about 100 as compared to titanium at about 25, and the magnesium is around 10.

Our plan is to fabricate samples which are about 5 cm in diameter and height. At least initially they will be in the shape of polygons with flat faces oriented around the spherical defect. Into these flat faces transducers will be bonded which will then sample the scattered energy ultimately coming from another transducer at one reference face.

TABLE III

Material Parameters*

MATRIX:

Ti		Ti-6%Aℓ-4%V	
ρ =	4.5 gm/cc	ρ =	4.42 gm/cc
v_ℓ =	6.07×10^5 cm/sec	v_ℓ =	6.34×10^5 cm/sec
v_s =	3.13×10^5 cm/sec	v_s =	3.03×10^5 cm/sec

SPHERE: Diameter: 400μ and 800μ.

(1) Cavity - Vacuum

(2) Tungsten Carbide - ρ = 13.8 gm/cc

v_ℓ = 6.66×10^5 cm/sec

v_s = 3.98×10^5 cm/sec

(3) Magnesium - ρ = 1.74 gm/cc

v_ℓ = 5.77×10^5 am/sec

v_s = 3.05×10^5 cm/sec

* ρ = density, v_ℓ = longitudinal wave velocity

v_s = shear wave velocity

With this experimental technique we hope to shed light on the scattering characteristics of spherical defects over the frequency range from 1 to 15 megahertz and make quantitative comparisons for both the frequency and the angular dependence that will be calculated from the theory.

I realize that in the practical NDT world, you are not dealing with nice single spheres, but rather complex geometries and a number of obstacles, but we feel that this is at least taking the first step to a detailed quantitative understanding of what goes on, with particular emphasis on mode conversion. Then, perhaps by using perturbation approaches, we can reduce some of these results to the more practical cases.

References:

1. C. F. Ying and Rohn Truell, J. Appl. Physics 27, 1086 (1956).

2. K. F. Herzfeld, Phil. Mag. 9, 741 (1930).

3. P. M. Morse and K. U. Ingend, Theoretical Acoustics (McGraw-Hill Book Co., New York, N.Y. 1968).

DISCUSSION

DR. PAUL PACKMAN (Vanderbilt University): In that acoustic energy pole figure, was that a continuous wave solution or pulsed wave solution?

DR. TITTMANN: Continuous wave.

DR. PACKMAN: Do you think you would get that drop again at 90 degrees for pulsed wave?

DR. TITTMANN: Probably not, if we are talking about very short pulses which carry with them a broad range of frequencies.

DR. PACKMAN: Thank you.

DR. GERALD GARDNER (Southwest Research Institute): For your experiments, do you plan to run them in a CW mode?

DR. TITTMANN: Essentially yes, because we are going to work with rather long pulses so that it can be viewed as being nearly monochromatic.

DR. GARDNER: You stated the size of the specimens, I gather, to have a radius of the order of three inches.

DR. TITTMANN: Two and a half inches in diameter.

DR. GARDNER: Diameter of three inches?

DR. TITTMANN: Right.

DR. GARDNER: Have you considered what is going to happen when you get acoustic waves sort of splashing around in there? If the pulse duration is pretty long, aren't you going to be bothered by a lot of complex, internal, multiple scattering?

DR. TITTMANN: The medium has a sufficiently high attenuation that the multiple scattering should be suppressed to tolerable levels, that is, levels not exceeding the scattering from grains and secondary phases.

PROF. MICHAEL FELIX (Purdue University): You mentioned the work of

Herzfeld, Ying, Truell and Morse in some of the theoretical development. I was wondering if you were familiar with the work of Black and Feshback who, as I recall, contributed significantly to some of the theoretical work in this area, particularly with regard to the case of various impedence mismatches on boundaries?

DR. TITTMANN: There are really a number of treatises out on the subject, and we decided to go to Herzfeld, Ying and Truell primarily because in their paper they developed a great deal of the physics going into an interpretation of the frequency dependence of the scattering.

PROF. ROBERT POND (Johns Hopkins University): I don't know that the oxide film between your surfaces makes any difference, but would you tell me again how you are going to get rid of these oxides in just your vacuum?

DR. TITTMANN: It is a combination of vacuum and heat treatment and if there is a metallurgist in the audience--

PROF. POND: That's me.

DR. TITTMANN: That's you? I see. Then I am telling the wrong person, but it is my understanding that oxides have two natures. Some of them can be pulled away rather easily with a vacuum and the others diffuse into the interior of the titanium alloy where they go into solid solution.

PROF. POND: Thank you.

MR. ROBERT CRANE (Air Force Materials Laboratory): Let me make a comment about that last question. Titanium is self-gettering, so it is easy to get rid of the surface oxides.

My question is on your third viewgraph or your third slide. You showed the hemispherical form and you said something about grain growth across the bond line, yet the slide was one with a hemispherical sample which obviously had not been polished, finely polished and etched. I was wondering, have you confirmed that the bonding process is sufficient to get grain growth?

DR. TITTMANN: Yes, we have some photographs of that; however, we don't show a hemispherical cavity in those particular photographs. (Note added in proof: Fig. 3 has been extended to include a micrograph of the grains across the bond line.)

MR. CRANE: Okay.

PROF. HENRY BERTONI (Polytechnic Institute of New York): How are you going to make a 400 micron spherical cavity and properly align it together with all the other problems?

DR. TITTMANN: Right. What we would do, of course, is machine each half of the sample, putting hemispheres into the center of each. Then we carefully align the edges of the two halves so that the two hemispheres will match. We have some practice with that, and our best effort now shows that the misalignment is down to about 40μ or 10%. I think we can do better than that.

PROF. JOHN TIEN (Henry Crumb School of Mines, Columbia University): Bernie, going back to the question about bonding, I can see how titanium, where it forms titanium oxide, either evaporates or it self-getters. Ti6Aℓ-4u, on the other hand, has aluminum in it.

DR. TITTMANN: Right.

DR. TIEN: What do you do with alumina, Al_2O_3, which may stay there and form other balls?

DR. TITTMANN: Will the Science Center metallurgist answer that?

DR. NEIL PATON (Science Center, Rockwell International): You are right, it does form alumina. It wouldn't be self-gettering, and it wouldn't dissociate in a vacuum, but you don't form alumina on Ti6Aℓ-4u.

DR. TIEN: I see, even though you have aluminum in there, it is not enough to give you a problem?

DR. PATON: That's right, in a nickel-based alloy, it would be enough, but not in this one.

DR. TITTMANN: The gentleman who is just speaking is Neil Paton, who is in charge of making these samples.

DR. OTTO GERICKE (U. S. Army Materials & Mechanical Research Center): Have you considered theoretically the case of a spherical surface discontinuity produced by drilling in from one side? Of course, you wouldn't have the wave trailing around the discontinuity, but you would have the cross-sectional surfaces exposed.

DR. TITTMANN: We have not considered that. That's an interesting point. We have considered the so-called flat-bottom hole, that is to say, a drilled hole with a flat bottom, and that gives a very interesting result which I can't discuss because of lack of time, but I can assure you the problem of the hemispherical void is very interesting. I think the situation would definitely be modified because the forward scattered wave plays an important role in the total reflection picture, so that would be an interesting question.